Worksheet 2

Name: Solutions Score: _____

1. Fill in the blanks in the table:

n vectors in $\mathbb{R}^m$	$m \times n$ Matrix	Linear $f : \mathbb{R}^n \to \mathbb{R}^m$
$\{v_1, \ldots, v_n\}$	$[v_1 \cdots v_n]$	$f(\vec{x}) = x_1\vec{v}_1 + \cdots + x_n\vec{v}_n$
linearly indep	No free var's in RREF	one to one
$\text{Span}(v_1, \ldots, v_n) = \mathbb{R}^m$	No zero rows in RREF	onto
Basis for $\mathbb{R}^m$	RREF is identity!	f isomorphism

   In the last row, $m = n$ because a basis $\{v_1, \ldots, v_n\}$ must have the same number of elements as the dimension m of $\mathbb{R}^m$.

2. When is a subset H of vectors in $\mathbb{R}^n$ a subspace? Give three examples of subspaces.

3. Give three ways of checking that a 3×3 matrix A is invertible.

4. Is the set of vectors linearly independent?

$$\begin{bmatrix} -10 \\ -5 \\ -1 \\ -9 \end{bmatrix}, \quad \begin{bmatrix} -10 \\ -8 \\ 6 \\ -3 \end{bmatrix}, \quad \begin{bmatrix} -5 \\ 5 \\ 4 \\ 1 \end{bmatrix}$$

   Yes

5. Does the set of vectors span $\mathbb{R}^3$?

$$\begin{bmatrix} -5 \\ 5 \\ -4 \end{bmatrix} \quad \begin{bmatrix} -3 \\ -4 \\ -1 \end{bmatrix} \quad \begin{bmatrix} -22 \\ 29 \\ -19 \end{bmatrix} \quad \begin{bmatrix} 19 \\ 2 \\ 11 \end{bmatrix}$$

No

6. Are the vectors linearly independent?

$$\begin{bmatrix}1\\1\\1\\4\end{bmatrix}, \quad \begin{bmatrix}2\\9\\2\\-3\end{bmatrix}, \quad \begin{bmatrix}-2\\-10\\2\\2\end{bmatrix},$$

Yes

7. Find the inverses:

(a)

$$\begin{bmatrix}-2 & 5\\-1 & 2\end{bmatrix}$$

$$A^{-1} = \begin{bmatrix}2 & -5\\1 & -2\end{bmatrix}$$

(b)

$$\begin{bmatrix}3 & 3 & -2\\1 & 0 & 4\\4 & 4 & -3\end{bmatrix}$$

$$A^{-1} = \begin{bmatrix}-16 & 1 & 12\\19 & -1 & -14\\4 & 0 & -3\end{bmatrix}$$

8. Find a basis for the Column space:

(a)

$$\begin{bmatrix}7 & -2 & 1 & -14\\9 & -3 & 0 & -24\\-15 & 5 & -1 & 37\end{bmatrix}$$

$$\begin{bmatrix}7\\9\\-15\end{bmatrix} \quad \begin{bmatrix}-2\\-3\\5\end{bmatrix} \quad \begin{bmatrix}1\\0\\-1\end{bmatrix}$$

(b)

$$\begin{bmatrix}-3 & -9 & -45 & -39\\-1 & -3 & -15 & -13\\3 & 6 & 33 & 27\end{bmatrix}$$

$$\begin{bmatrix}-3\\-1\\3\end{bmatrix} \quad \begin{bmatrix}-9\\-3\\6\end{bmatrix}$$

9. Find a basis for the null space

(a)

$$\begin{bmatrix} -2 & 0 & -2 \\ 26 & 5 & 6 \end{bmatrix}$$

Reduced Row Echelon Form:

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -4 \end{bmatrix}$$

Basis:

$$\begin{bmatrix} -1 \\ 4 \\ 1 \end{bmatrix},$$

(b)

$$\begin{bmatrix} -1 & 3 & -1 & -4 \\ 0 & 0 & 1 & 4 \\ -3 & 9 & 7 & 28 \end{bmatrix}$$

Reduced Row Echelon Form:

$$\begin{bmatrix} 1 & -3 & 0 & 0 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Basis:

$$\begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} 0 \\ 0 \\ -4 \\ 1 \end{bmatrix},$$